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A NOTE ON THE CALCULATION OF FLOW FRICTION IN CHANNELS WITH AND WITHOUT SEPARATED FLOW

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The theoretical determination of total pressure losses and other aerodynamic parameters in two-dimensional and axisymmetric channels is often difficult. In connection with the recent appearance of certain misleading papers on this subject and the existing confusion of terminology, further discussion is very desirable.

In the two preceding notes two main questions were raised: the use of methods of boundary layer theory to calculate hydraulic losses in unseparated diffuser flow and the possibility of dividing losses in diffusers with separated flow into "friction losses" and "expansion losses" using generalized empirical relations.

Zaryankin is right in criticizing the lack of a physical basis for dividing the losses in diffusers into two components This was clear even to the author of the method, Idelchik, who stressed that the separation was arbitrary [1]. It is difficult to agree with Zaryankin, however, when he rejects the approximate engineering method, without offering anything in its place other than the statement that a solution of the problem is desirable "on the basis of the general concepts of the aerodynamics of the mechanism of losses" [26].

The status of both issues is correctly outlined in [27], where a sound review of Zaryankin's position is offered.

Let us discuss the above points in more detail.

An arbitrary division of the total pressure losses in diffusers into two components ("friction losses" and "expansion losses") was proposed by Idelchik in deriving an engineering method for calculating the losses in diffusers for all possible values of the expansion angle [1]. Since the so-called "friction losses" are very small at comparatively large expansion angles, while the "expansion losses" are determined on the basis of a generalization of the experimental data, it is natural that Idelchik's proposed interpolation formulas for not very small diffuser expansion angles should give results close to reality.

Both terms – "friction losses" and "expansion losses" – are very imprecise. If, in using the term "friction losses," we have in mind friction within the fluid, then the term "expansion losses" loses its meaning, since all the losses stem from viscosity of the fluid, i.e., friction. Since the quantitative determination of the friction losses involves calculating the friction force at the walls of the diffuser channel, the question arises whether the friction losses may not be reduced to friction at the walls. In this case it would be wrong to assert that in diffusers without separation only friction losses occur. Indeed, in unseparated flow of a fluid in a diffuser, the total pressure losses are due both to fluid friction at the diffuser walls and to deformation of the velocity field in cross sections of the diffuser ("expansion losses").

This remark will become clear enough after examination of the possible patterns of unseparated flow in channels with straight axes. This is especially desirable in that in both notes, in speaking of the application of the methods of boundary layer theory to the calculation of flow in diffusers, the authors have in mind only one particular flow system, namely, flow with a potential core. However, a much wider class of unseparated flows may be studied by the methods of boundary layer theory.

Linear stabilized flow in channels of constant cross section. It is known that this flow becomes steady at quite large distances from the inlet. The dynamic pressure at individual cross sections of the tube is then constant, as a result of which the total pressure losses are wholly determined by the static pressure drop along the flow. Only in this special case are the hydraulic losses in a tube uniquely associated with the friction coefficient at the wall. Thus, for example, in the case of an annular tube (inside radius r_2, outside r_1), the following relation holds [3]:

$$\frac{c_{f1}+\xi c_{f2}}{1+\xi}=-\frac{dp}{dx}\frac{r_1-r_2}{\rho u_\delta^2}\quad\left(\xi=\frac{r_2}{r_1}\right), \tag{1}$$

where c_{f1} and c_{f2} are the friction coefficients at the two surfaces $[C_{f1}=\tau_{\omega 1}/(\rho u_\delta^2/2),\ C_{f2}=\tau_{\omega 2}/(\rho u_\delta^2/2)]$, u_δ is the maximum velocity in the section, and p is the pressure.

In the particular cases of a circular (ξ = 0) and a two-dimensional (ξ = 1) tube, Eq. (1) may be simplified:

$$\begin{aligned} c_f &= -\frac{dp}{dx}\frac{r}{\rho u_\delta^2}\quad (\xi=0,\quad r_2=0),\\ c_f &= -\frac{dp}{dx}\frac{h}{\rho u_\delta^2}\quad (\xi=1,\quad h=r_1-r_2). \end{aligned} \tag{2}$$

In the two other flow systems examined below, there is no direct relationship between the hydraulic losses and the friction coefficient at the wall, since these two characteristics (force and energy) are determined from two different equations – the momentum and energy equations.

Stabilized flow in a curved channel. It is assumed here that even in the initial section of the channel the boundary layers at the walls are completely fused, so that at all sections the boundary layer thickness is equal to half the width in the case of a two-dimensional channel or to the radius in the case of an axisymmetric channel. Flow of this kind in two-dimensional or axisymmetric channels may be computed using the integral momentum relation

$$\frac{d\delta^{**}}{dx}+\frac{u_\delta^1}{u_\delta}(2\delta^{**}+\delta^*)-\left(\frac{u_\delta^1}{u_\delta}+\frac{1}{\rho u_\delta^2}\frac{dp}{dx}\right)\delta=\frac{1}{2}c_f, \tag{3}$$

where δ, δ^*, δ^{**} are, respectively, the boundary layer thickness (area), and the displacement and momentum thickness.

The investigation is somewhat simplified in the particular case of channels with straight walls. For example, in the case of a two-dimensional channel with straight walls, linear stabilized flow is hydrodynamically possible (flow in a dihedral angle from a plane source at the vertex). Such flow has, of course, similarity of the velocity profiles at all cross sections of the channel and, as a result, a constant friction coefficient along the walls. It therefore follows from the condition of constant flow in cross sections $Q = c_1\delta u_\delta$ that the Re number = $\delta u_\delta/\nu$ = const. Radial flow of this kind in two-dimensional channels has been thoroughly investigated both for laminar (exact solution of the Navier-Stokes equations) and for turbulent flow [4, 5]. For radial flow in a two-dimensional channel, integral relation (3) takes the form [5]

$$(1-H^*-H^{**})\operatorname{tg}\frac{\alpha}{2}=\frac{1}{2}(\lambda'+c_f), \tag{4}$$

where the quantities $c_f=\dfrac{\tau_w}{\frac{1}{2}\rho u_\delta^2}$, $\lambda'=\dfrac{h}{\rho u_\delta^2}\dfrac{dp}{dx}$, $H^*=\dfrac{\delta^*}{\delta}$, $H^{**}=\dfrac{\delta^{**}}{\delta}$ depend on the expansion angle of the channel and the Re number $Re_h = hu_\delta/\nu$ and remain unchanged along the flow, α is the total expansion angle, and h is the width of the channel (h = 2δ).

When α = 0, expression (4) clearly transforms into (2). It is interesting to note that the total pressure loss factor for any part of such a channel is characterized by the expansion ratio n in accordance with the formula

$$\zeta=\zeta_0(1-1/n^2), \tag{5}$$

where

$$\zeta_0=\frac{1-H^*-H^{***}}{1-H^*}-\frac{1}{2}\lambda'\operatorname{ctg}\frac{\alpha}{2},\quad H^{***}=\frac{\delta^{***}}{\delta}$$

and δ^{***} is the energy thickness, while ζ_0 depends only on the expansion angle and Re_h. For a maximum expansion angle $\alpha = \alpha_{max}$, when at all sections of the diffuser the turbulent boundary layer is in the preseparated state (c_f = 0), the coefficient ζ_0 = 0.1 [5] at all Reynolds numbers. Thus, even when friction at the walls is absent, the "friction losses" are given by Eq. (5), which coincides with Eq. (3) of the first note and with Eq. (1) of the second.

In real conditions, radial flow in a two-dimensional diffuser can become established only at some distance from the initial section. Such flow was first studied experimentally by Dönch [6] and Nikuradse [7], who showed that, for a fixed Re number, all the flow properties are essentially determined by a single geometric parameter, the expansion angle. Perhaps this is the source of the erroneous and, unfortunately, widely held opinion that in all flow cases the expansion angle is the single important geometric parameter of a diffuser.

In the theoretical investigation of stabilized turbulent flow in conical diffusers with small expansion angles the hypothesis that the flow is radial is often taken as a starting point, and all streamlines are assumed to be straight lines starting from the vertex of the cone [8-10]. It should be noted, however, that, in contrast with the corresponding flow in a two-dimensional diffuser, radial flow in a conical diffuser is hydrodynamically impossible, because here the conditions of constant flow along the diffuser $(Q \sim u_\delta r_1^2)$ and constant Reynolds number $(\mathrm{Re} = r_1 u_\delta / \nu)$ are not simultaneously fulfilled. This applies equally to the case of laminar flow studied by Slezkin [11].

Flow in channels with a potential core. In this case, when the channel is preceded by a tube of no great length and the boundary layers formed on its walls do not fuse at the exit section, the flow outside the boundary layer on the walls of the channel (diffuser), at any point up to the section where the boundary layers meet or the flow separates, may be assumed to be potential. In a number of cases it is assumed that at the initial section of the channel the velocity distribution is uniform. In practice, an almost uniform initial velocity distribution may be achieved by mounting a smooth collector at the entrance. For uniform initial velocity distribution the length of the section of unseparated flow in the diffuser is a maximum.

Modern methods of the theory of laminar and turbulent boundary layers may be widely used to solve problems thus formulated. There are many papers devoted to calculating flows in the inlet sections of annular, two-dimensional, and circular tubes and in two-dimensional and axisymmetric diffusers [12-15].

When a potential core exists, in the latter case the Bernoulli equation is valid, and the third term on the left of Eq. (3) vanishes. Then the total pressure loss factor is expressed by the formula [14]:

$$\zeta = \frac{\Delta^{***}}{n^2 (1 - \Delta^*)^3} - \frac{\Delta_0^{***}}{(1 - \Delta_0^*)^3}, \qquad (6)$$

where Δ^{***} and Δ^* are the ratios of the energy area and the displacement area to the cross-sectional area, and the subscript "0" corresponds to the initial section of the channel. Using Eq. (6), we can investigate the influence of the expansion ratio, expansion angle, and initial flow nonuniformity on the loss factor for two-dimensional and axisymmetric diffusers. Calculations show that, for a given Re number and flow nonuniformity in the initial section, the losses in a channel with straight walls depend on two parameters – the expansion angle and the expansion ratio. Furthermore, with increase of expansion angle in a diffuser of given relative length, the total pressure loss factor decreases until the increasing expansion angle causes separation of the boundary layer.

The formula given above is known to have been extended to the case of flow of a compressible gas with and without heat transfer between the gas and the channel walls [16, 17]. Essentially similar formulas are widely used to calculate the pressure losses in the curved interblade channels of turbine cascades [18-20].

It should be noted that no sharp demarcation line exists between the flow systems enumerated above.

For example, in the same channel, flow with a potential core may exist near the initial section, and after fusion of the boundary layers established flow.

Although the methods of boundary layer theory are now widely applied in the investigation of unseparated flow in channels, the situation is much less satisfactory as regards the theoretical determination of total pressure losses in diffusers with flow separation. In spite of isolated successes in investigating separated flow [21], the problem is far from resolved, and the most reliable calculations of the loss factor in diffusers with separated flow are still based on generalized experimental relationships. Of course, in establishing such generalized relations, one should strive to take into account not only geometric but also aerodynamic flow parameters and to consider to some extent the prehistory of the flow.

Here the most reliable results are obtained on the basis of systematic experimental investigations. For example, Kmonicek [22] has derived interpolation formulas for the loss factor from careful measurements of the flow parameters in a series of conical diffusers with a wide range of variation of the expansion angle (4°30'-28°) and expansion ratio (1.6-16) and various initial flow nonuniformities.

In a number of cases, however, it is necessary to determine without experiments the flow friction of diffusers of complex shape with deliberately detached flow: noncircular cross section, curved centerline, etc. For such purposes the so-called method of equivalent plane and conical diffusers has been widely used [2, 23]. It is usual to call a plane or conical diffuser equivalent to a given diffuser of complex shape, if the centerline and the areas of initial and end sections are the same.

It then turns out that in a number of cases, using the parameters of the equivalent diffuser – expansion ratio n_{eq} and expansion angle α_{eq} – one can reduce the results of experimental investigations of diffusers of different shape to single "universal" relations [24, 25].

The faults of this method become especially evident in the case of diffusers with strongly curved axes or walls, since the parameters of the equivalent diffuser fall a long way short of taking fully into account the effect of curvature of the walls or axis.

A sounder method for curved diffusers is that based on the concept of the local expansion angle of a curved diffuser, which has been widely used by the authors of [2]. This method allows the influence of the shape of the elements of the diffuser on the losses to be taken into account to any degree, and also makes the determination of the equivalent diffuser more accurate. The method has proved useful in constructing the aerodynamic contour of radial-annular diffusers [2, 24], enables qualitatively correct results to be obtained, and is in satisfactory agreement with experiment.

However, in some cases the calculated values of the losses in radial-annular diffusers differed noticeably from the experimental values.

Methods of this kind will clearly find wide application until a sounder method of calculating the flow friction in diffuser channels with separated flow is found.

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